Simulation of characteristics and artificial neural network modeling of electroencephalograph time series

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Electroencephalograph data of normal individuals with their eyes closed is analyzed and modeled. The characteristics of the observed time series correspond to a filtered Gaussian random noise model modified with a suitable falloff in the power spectrum. An artificial neural, network model produces an excellent fit to the relatively short time series data without any hidden neurons, implying that the underlying system is predominantly deterministic and linear. [S1063-651X(97)01104-5]

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There has been considerable interest in applying methods of nonlinear dynamics to the electroencephalograph (EEG) data [1]. In order to understand the nature and dynamics of the underlying system various characteristics [2] of the EEG time series such as power spectrum, correlation dimension, embedding dimension, Lyapunov exponent, surrogate data study, deterministic versus stochastic (DVS) plot, structure function, etc. have been investigated. The conclusions drawn from these studies are not completely reliable because the EEG time series is most often short and noisy [1]. All the same, there appears to be general agreement that for normal individuals in resting state (eyes closed) the underlying system is of high dimension and does not exhibit low dimension deterministic chaotic behavior. Also, the questions whether the system is linear or nonlinear and deterministic or stochastic are not altogether settled [1].

In this paper, we study the eyes closed electroencephalograph (ECEEG) time series with special emphasis on simulation. The idea is to construct a model time series that would simulate the observed ECEEG characteristics. This would enable one to arrive at more definite conclusions about the nature of the ECEEG data. More precisely, we first consider a filtered time series generated from a Gaussian random noise model and compare the characteristics with those obtained with the ECEEG data. We find that, except for the autocorrelation function (ACF), the power spectrum, and the embedding dimension, the characteristics we studied are very similar for the two systems. We then suitably modify the power spectrum of the stochastic (Gaussian random noise) model and show that it is able to qualitatively simulate all the characteristics of the ECEEG data including the ACF and the embedding dimension.

On the basis of our studies we conclude that the ECEEG data we analyzed are weakly nonlinear and deterministic. In spite of the existence of nonlinearities the DVS plots for the relatively short time series do not reflect any such behavior. This is also confirmed by carrying out an artificial neural network (ANN) modeling of the time series, where we find an excellent fit with no neurons in the hidden layer.

We next give some details of our analysis, simulation studies, and ANN modeling. As regards the data [3], the EEG data were collected from the 8 loci of the international 10–20 system using a conventional EEG machine (Neurofax, Nihon Khoden) coupled to a 486 PC-AT system with analog to digital converters (DT-2841) and array processors (DT-7020) of Data Translation Inc. Four normal male subjects having no history of neurologic or psychiatric disorders participated (mean age 28.5, standard deviation 3.25, range 22–35). Subjects were tested in the morning in a soundproof, electrically shielded room, while sitting on a comfortable chair. Silver cup electrodes were attached to the 8 scalp loci ($F3$, $T3$, $P3$, $O1$—reference electrode at $A1$; $F4$, $T4$, $P4$, $O2$—reference electrode $A2$ and the forehead ground electrode) for monopolar recording. They were instructed to be in a relaxed state with their eyes open for 5 min and then closed for 5 min. On-line digital recording continued for 30–45 minutes for each subject and the procedure was repeated four times on the same subject. The EEG signals were digitized at 256 samples/channel/s to the PC-AT and later ported to an HP-9000/735 Graphics Workstation. The signals were filtered through a bandpass filter (0.5–32 Hz, fourth order Butterworth twice cascaded) after subjecting them to baseline corrections. Data of each subject are visually screened to obtain artifact free data of at least 8 s duration. The analysis was conducted in the EEG segments from the $F3$ frontal channel, which may not contain strong α, but we clearly see it in the posterior lead $P4$ records.

We analyzed a number of different ECEEG time series consisting of 4096 data points from channel $F3$. The data sets were taken from four people, and from each individual's record we included a number of stretches of 4096 points. The results we report here are for one record but they are representative and characteristic of all the $F3$ (eyes closed) records we analyzed. Thus, we expect these results to be valid for normal people in the resting state with their eyes closed. We used the method of average mutual information for determining the lag time τ [4] and the method of false nearest neighbors [4] to obtain the embedding dimension d. In actual calculations, τ is the first minimum in the plot of average mutual information against time lag. For this value of τ, we take d to be the first zero in the plot of the number of false nearest neighbors versus dimension. We also verify that this zero is followed by all zeros (or small values) for higher values of dimension. In a typical case we get $\tau = 9$ and $d = 12$. Figure 1 shows the power spectrum, the number

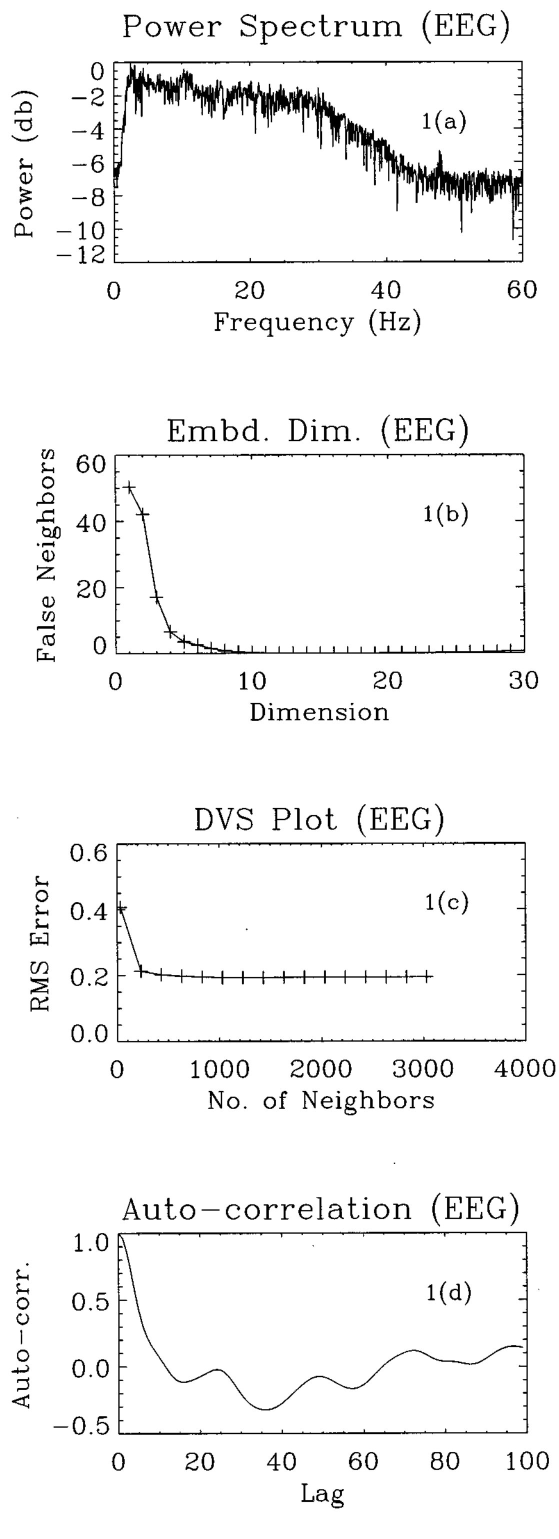

FIG. 1. Characteristics of filtered ECEEG data.

of false nearest neighbors against dimension, the DVS plot [5], and the ACF. We observe [Fig. 1(a)] that the power spectrum is broadband and falls off with frequency as expected from a nonlinear system [6]. The falloff can be fitted equally well by either an exponential or a power law, and hence from this result we are unable to say if the nonlinearities are due to the deterministic or stochastic nature of the system. The value of embedding dimension $d = 12$ [Fig. 1(b)] indicates that one has a high dimension system that is very likely deterministic. The DVS plot [Fig. 1(c)] shows that the rms error falls rapidly by a factor of about 2 as we increase the number of neighbors N_{nn}, and has a minimum for $N_{nn} \approx 250$. The initial fall occurs because of the effect of fluctuations in the data. This effect dominates for a small number of neighbors and decreases with an increase in N_{nn}. The fact that the minimum is away from small N_{nn} implies that the system is certainly not a low dimension chaotic system. The near constancy beyond $N_{nn} \approx 250$ suggests that a global linear model is most likely very good. The autocorrelation function [Fig. 1(d)] exhibits long time coherence and hence deterministic behavior [7]. We also evaluated the largest Lyapunov exponent using Wolf's [8] method and find the value to be 0.0651. This indicates that the nonlinearity (if any) is weak. Of course for the short noisy series that we have, the result cannot be very reliable. In order to test these findings we generate a time series from a Gaussian random noise model and apply the same filter as we did to the ECEEG data. Again we determine the characteristics of this model [filtered Gaussian random noise (FGN)] series. The results are shown in Fig. 2. Note that there is no falloff in the power spectrum. The number of false nearest neighbors rises after going through a minimum and in the DVS plot the rms error falls by only 20%, and remains constant at a relatively large value. The autocorrelation function also falls sharply and then oscillates about zero. Thus, it is clear that ECEEG data are certainly not compatible with the FGN model as implied by Albano and Rapp [9] in their simulation. It ought to be stressed that the result is valid for all frequencies allowed by the band pass filter (0.25–32 Hz) and not only for the α activity.

To further clarify these issues we modified the filtered Gaussian random noise time series by artificially providing an exponential falloff $e^{-\alpha f}(\alpha=0.23)$ in the power spectrum. This model, called the simulated EEG (SEEG) model, gives a falloff of $\sim 10^{-3}$ at $f \sim 30$ cycles/s. The characteristics of this time series are shown in Fig. 3. We observe that this simulated times series has the same qualitative features as the ECEEG data shown in Fig. 1. Similar qualitative results are also obtained by providing a power law $(1/f^2)$ falloff in the power spectrum of the filtered Gaussian random noise model.

We also carried out an ANN study of the ECEEG time series, with and without neurons in the hidden layer. The number of neurons in the input and output layers were $n_1 = 12$ and $n_3 = 1$, respectively. When a hidden layer was present the number of neurons in it was $n_2 = 8$. The transfer function was hyperbolic tangent for the hidden layer and linear for the output layer. The network is trained using the conventional back propagation method [2]. It ought to be recalled that without the hidden layer one is attempting to model the time series according to

$$x_{t+1} = \sum_{k=1}^{d} a_k x_{t-(k-1)\tau}. \tag{1}$$

When one includes a hidden layer the model is given by

$$x_{t+1} = f(x_t, x_{t-\tau}, \ldots, x_{t-(d-1)\tau}). \tag{2}$$

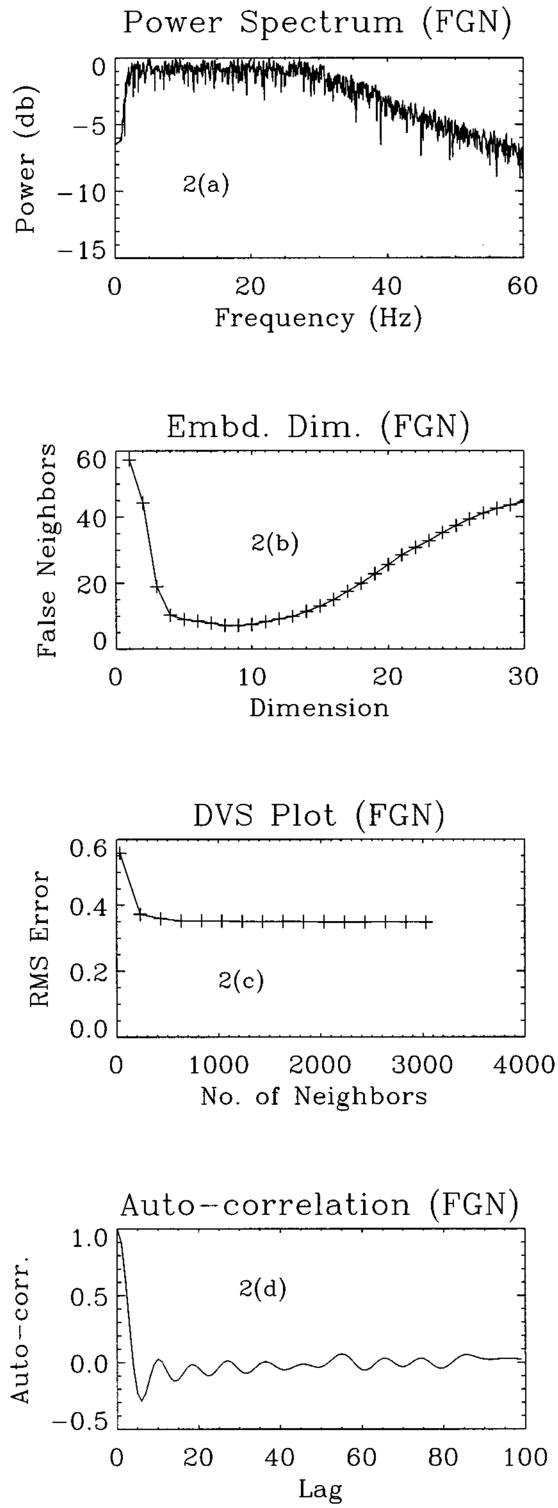

FIG. 2. Characteristics of filtered Gaussian random noise (FGN) time series.

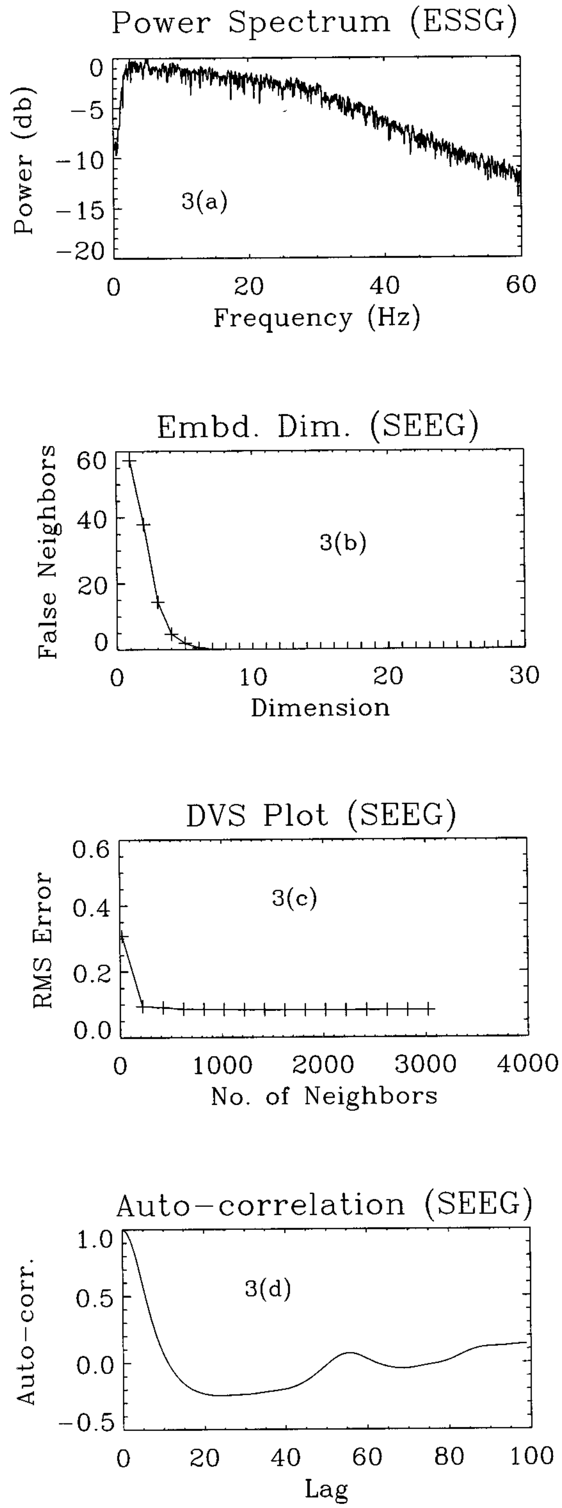

FIG. 3. Characteristics of simulated EEG (SEEG) time series.

The measure for the quality of fit is defined by average relative variance (Φ_{ARV})

$$\Phi_{\mathrm{ARV}}=\frac{1/N\Sigma_{i=1}^{N}E_i^2}{(\text{ variance of } N \text{ data points})}, \tag{3}$$

where E_i is the error difference between the ith observed and calculated values. The network was trained using the first 3200 data points. The remaining 896 data points (test set) were used to evaluate I_{ARV} [Eq. (3)]. The main result of the ANN modeling of the ECEEG data is that the quality of the fit is essentially the same for a linear (no hidden neurons; $\Phi_{\mathrm{ARV}}=0.0703$) and a nonlinear (with hidden neurons; $\Phi_{\mathrm{ARV}}=0.0705$) network. These results show that the ECEEG series can be very nicely fitted by a pure linear autoregressive model. A comparison of the results of calculation of the ANN model with the test data set is shown in Fig. 4. Note

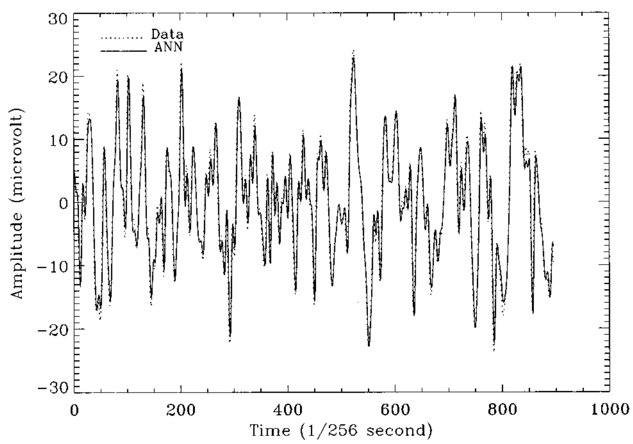

FIG. 4. Comparison of calculation of ANN model with the test data set.

that Paluš [10] has concluded from his analysis of the EEG data that the system is linear and stochastic. We find that it is linear and deterministic. As an interesting aside we find that the SEEG series can also be very well fitted by a linear ANN model.

In summary, we find the following features in the ECEEG data that we analyzed. (i) ECEEG data *cannot* be understood in terms of a purely filtered Gaussian random noise model. (ii) From the power spectrum, the data exhibit nonlinearities (weak) but from the falloff in frequency it has not been possible for us to conclude if it is due to deterministic chaos or stochastic dynamics. The nonlinear behavior is not seen in DVS plots. (iii) The embedding dimension is high. (iv) From the DVS plot and ANN modeling we clearly find that the system is linear.

We emphasize that these results refer only to the eyes-closed data. Under the condition of eyes open or providing sensory stimuli the analysis reveals quite different features. Finally, in view of these findings we conjecture that in the present case the falloff in the power spectrum ought to be exponential, indicating that the system is weakly nonlinear and weakly chaotic, with high embedding dimension.

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